Over-the-air Function Computation in Sensor Networks

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Abstract – Many sensor applications are interested in computing a function over measurements (e.g., sum, average, max) as opposed to collecting all sensor data. Today, such data aggregation is done in a cluster-head. Sensor nodes transmit their values sequentially to a cluster-head node, which calculates the aggregation function and forwards it to the base station. In contrast, this paper explores the possibility of computing a desired function over the air. We devise a solution that enables sensors to transmit coherently over the wireless medium so that the cluster-head directly receives the value of the desired function. We present analysis and preliminary results that demonstrate that such a design yield a large improvement in network throughput.

1. INTRODUCTION

The last decade has seen significant advances in sensor technologies and protocols, which has led to interest in large sensor deployments for monitoring various physical quantities such as temperature, pressure, humidity, *etc.* [29, 9]. Deployments of over 1000 nodes are emerging quickly and expected to become common [14]. They are used for environmental monitoring, seismic sensing, factory automation, and process control. As the density and scale of sensor networks grows, it becomes increasingly important to come up with data transmission schemes that use the medium efficiently.

In many sensor applications, there is interest only in summary properties of the data –such as average, sum, maximum, minimum, *etc.* – and there is no need for continuously collecting all sensor values [13, 17, 12, 18]. For example, the application may be interested in computing the average temperature in an area, or checking that the maximum temperature does not exceed a threshold. For such applications, transmitting all data to the base station is inefficient. To address this issue, much past work in sensor networks has advocated data aggregation [13, 17, 12, 18]. These schemes typically aggregate the measurements locally at intermediate nodes. For example, the network may be divided into clusters. One node is elected as a cluster-head. The other nodes in the cluster transmit their data sequentially to the cluster-head, which computes the desired function and forwards it to the base station. While this improves the overall network throughput, the sequential transmissions of sensor data to the cluster-head still consumes significant wireless bandwidth.

In this paper, we ask if it is possible to aggregate the data over the air, directly delivering the desired function. For example, can sensors intelligently modify the data that they transmit, such that when they transmit jointly, the cluster-head simply receives the result of the function? Such an approach can yield a large reduction in bandwidth consumption. Specifically, it ensures that bandwidth needs do not increase with the size and density of the network; rather they stay limited by the number of bits in the desired function.

We propose CompAir, a novel technique to aggregate data over the air. CompAir leverages the fact that the wireless channel can combine signals in both a linear and non-linear manner. Specifically, when signals are transmitted concurrently, the wireless channel naturally produces a linear combination of their values weighted by the channels' coefficients. This allows us to manipulate the transmissions to produce a variety of linear functions such as sum, average, variance, *etc.* The wireless channel can also be used to compute the non-linear "OR" function, by having the receiver observe whether there is any power on the channel (beyond the typical noise level). We show that this property can be used as a primitive to compute more complex non-linear functions such as minimum, maximum, and median.

The paper further analyzes the robustness of the computed functions to channel noise and presents preliminary results from a USRP testbed that demonstrate that such a design can yield dramatic bandwidth savings in large sensor networks.

2. RELATED WORK

Related work falls in three categories.

(a) Aggregated Wireless Sensor Networks : Most data aggregation work can be classified as: tree-based [17, 26, 16, 13], cluster-based [12, 27, 30, 18] or multipath [20, 19, 7]. At a high level, these schemes works as follows. Nodes transmit their values sequentially to a cluster-head or parent using unicast transmissions and a scheduling mechanism, say TDMA, to avoid collisions. The parent/cluster-head computes the aggregated function and then transmits this aggregate upstream. In contrast, in CompAir, all sensors transmit their data concurrently, and the parent/cluster-head simply receives a single value corresponding to the aggregate function. This can improve spectrum efficiency and data latency.

(b) Collision Decoding: Our work is motivated by recent wireless trends advocating concurrent transmissions and collision decoding. Systems like ZigZag [15], Buzz [25], Caraoke [2], and compressive sensing [4, 6], treat wireless collisions as a code and decode the bits despite interference. Other systems like MegaMIMO and AirSync [21, 5] exploit coherent transmissions from multiple nodes to eliminate interference. Past systems however are interested in decoding all of the original bits. In contrast, CompAir notes that, in many cases, the objective is to compute a function over the bits, and devises a mechanism for computing such functions over the air.

(c) Information Theoretical Approaches to Distributed Function Computation: There has been a lot of recent interest in distributed function computation from the information theory community [10, 28, 3, 22, 11]. This work either does not take advantage of the properties of the wireless channel and simply focuses on minimizing communication cost over a wired network like the Internet, or does not exploit the ability of nodes to transmit jointly in a coherent manner. Further, this work is theoretical. In contrast, CompAir fundamentally leverages the ability of wireless sensors to transmit coherently to compute a variety of functions, and has been prototyped in a testbed of USRPs.

3. DESIGN SCOPE

CompAir is a wireless system that allows a collection of sensors to transmit their data concurrently such that the receiver receives over the medium a function of the sensors' data, such as sum, max, min, etc.

Before delving into the details, we clarify the scope of this particular paper and assumptions underlying CompAir.

- CompAir is targeted towards *large and dense* sensor networks, which incur a high overhead from collecting individual sensor measurements from all the sensors, and can therefore obtain significant benefit from over-the-air aggregation of these measurements.
- We describe how CompAir works in the context of aggregating data from multiple sensors at a single cluster-head. The approach naturally extends to a multi-level hierarchy, including aggregating data from multiple cluster-heads.
- In this paper, we assume that sensors can transmit their data coherently (*i.e.*, synchronized in time and phase). Sensors can do so using recently developed synchronization techniques such as AirShare [1]. AirShare is a simple low-overhead system that synchronizes nodes by transmitting the reference clock over the air, providing a tool for generic distributed PHY protocols. We refer the reader to [1] for further details regarding AirShare.

The next two sections explain the basic idea underlying over-the-air function computation. We start with linear functions, then extend the design to non-linear functions.

4. COMPUTING LINEAR FUNCTIONS

It is well known that the wireless channel can be modeled as a linear system. CompAir uses this property to compute linear functions over sensor measurements. We will start by explaining how to compute the sum of the sensors' measurements. We then extend CompAir to other linear functions.

If multiple sensors transmit their signals simultaneously and coherently, the cluster-head receives a linear combination of these signals. Specifically, let sensor i, $1 \leq i \leq N$ (N is the number of sensors), transmit the signal x_i, and let the channel from sensor i to the cluster-head be h_i. Then, the received signal at the cluster-head is $R = \sum_{i=1}^{N} h_i x_i$. (For simplicity of exposition, we omit the noise terms from the equations in this section. §6 describes in detail the effect of noise, and how CompAir is robust against it.)

In order for the cluster-head to receive the true sum of the values x_i, each sensor needs to compensate for its channel. In particular, each sensor needs to transmit a value

$$z_i = \frac{x_i}{h_i} \tag{1}$$

such that:

$$R = \sum_{i=1}^{N} h_i z_i = \sum_{i=1}^{N} x_i. \tag{2}$$

Naively, computing the channel from each sensor to the cluster-head would require each sensor to individually send a training signal to the cluster-head, and for the cluster-head to then transmit the measured channel to the sensor. Such a system would incur high overhead, and negate the benefits of using CompAir!

Instead, CompAir uses channel reciprocity to allow each sensor to measure its channel to the cluster head with very low overhead (i.e., it uses the fact that the forward and reverse channels are always the same up to a constant multiplier due to differences in hardware between the transmit and receive chains [23].) As mentioned in §3, CompAir synchronizes the oscillators on the sensors and the cluster-head using distributed synchronization mechanisms, particularly AirShare [1]. Once the oscillators are synchronized, each CompAir sensor performs a one-time calibration of the channels to and from the cluster-head to determine a calibration factor $K_i = \frac{h_i(0)}{g_i(0)}$, where $h_i(0)$ is the initial channel (at the time of calibration) from sensor i to the cluster-head as described earlier, and $g_i(0)$ is the corresponding channel from the cluster-head to the sensor. Note that this calibration factor depends only on the hardware on the nodes, and needs to be repeated only infrequently to update the calibration factor.

Each aggregated transmission is initiated by a request packet from the cluster-head. Each sensor measures the channel $g_i(t)$ *from* the cluster-head using the request packet. It can then compute its channel $h_i(t)$ *to* the cluster-head as $h_i(t) = K_i g_i(t)$. It then substitutes this computed channel in Eq. 1 to determine the transmitted values z_i. Note that each sensor needs to know only its own channel value, and hence this computation can be done completely locally. Further, the ongoing channel measurement overhead is small and constant independent of the number of sensors.

4.1 Computing Other Functions

Now that we have described how CompAir can compute the sum of transmitted values over the air, we can extend it to a variety of other functions, which are either linear or can be reduced to linear computations.

Arithmetic Mean: The AM can be computed as the sum divided by the total number of sensors.

Geometric Mean: The GM is itself not linear, but can be reduced to linear computations over the air. Specifically, the logarithm of the GM is simply the AM of the logarithms of the original values. Therefore, each node transmits the log of its value, and the cluster-head computes the AM as above. It can then compute the antilog to recover the actual geometric mean. The same idea can be used to compute the product of observed values, rather than the sum.

Weighted average: Each sensor simply sends $w_i x_i$ where w_i is the weight associated with that sensor.

Count (predicate): Suppose the system wants to count the number of sensors whose readings satisfy a certain value, say, if the temperature exceeds a threshold T. In such a case, the cluster head sends the corresponding predicate ("temperature > T") in its request. Every sensor evaluates the predicate locally, and if it is true, it sends the value $x_i = 1$. The cluster head then simply receives the sum of these values. Note that this function can also be used to count the total number of sensors by simply setting the predicate to *true*.

Variance: The variance can be computed as $E(x^2) - E^2(x)$, where E(.) is the expectation/mean function. The system can use CompAir to compute the mean of x_i^2 (each sensor transmits x_i^2), as well as the mean of x_i. The cluster head can use these values to determine the variance.

Regression: There are several applications that involve the cluster head determining the distribution of the sensor measurements. For instance, the cluster head might desire to determine the best linear fit to the observed measurements when plotted against the coordinates of the sensor. The coefficients of such a fit can be measured simply by each sensor transmitting the relevant terms of the linear regression. For instance, if the best linear fit of the observed values can be written in the form $y = \alpha + \beta x$, and each sensor's location and observation value pair is (x_i, y_i), then we can simply compute the following values: $E(xy)$, $E(x)$, $E(y)$, $E(x^2)$ over the air. The cluster head can then compute the best fit as $\beta = \frac{E(xy) - E(x)E(y)}{E(x^2) - E^2(x)}$ and $\alpha = E(y) - \beta E(x)$.

5. GOING BEYOND LINEAR FUNCTIONS

In this section, we extend CompAir to compute some non-linear functions such as maximum and minimum. The basic idea with these functions is to leverage the fact that the wireless channel also effectively provides an OR function. In particular, it can differentiate the case of when power is being transmitted on the medium by one or more devices, from the case of no power being transmitted by any device.

Let us start by explaining how we compute the maximum. We can combine the bit representation of the data with the channel's "OR" function to compute the maximum value across the sensors as follows. Computing the maximum proceeds in rounds, from the high order bit to the low order bit. In the first round, every node that has a 1 in the MSB transmits, while nodes that do not have a 1 in that bit stay silent. The cluster-head then detects whether power is received in that round. If so, it determines that there is at least one node in the network that has a 1 in the MSB, and sets the MSB of its computed maximum to 1. In the next round, it then sends a request asking only nodes that had a 1 in the MSB to transmit, if they have a 1 in the second most significant bit. If no nodes transmit in this round, the cluster head determines that the second most significant bit of the maximum is 0. The cluster head then initiates the third round, and so on, till it has computed the last bit. In this manner, the cluster head can determine the maximum value across all sensors in the network. Note that the number of rounds in this computation is determined only by the bit resolution of the measured values, typically 8-12. This is significantly less than the number of sensors transmitting to the cluster-head in dense sensor networks. Further, note that unlike in the case of linear functions, CompAir sensors need not compensate for the amplitude of the channel to ensure correct computation of the "OR" function.

Similar to the maximum, CompAir can also compute the minimum. It does so by computing the maximum of the one's complement of the measured values. The one's complement, v, of an n-bit value x, is $2^n - x - 1$. Thus, computing the minimum over the x is the same as computing the maximum over the v. Further, v can be obtained from x simply by complementing all the bits in the representation of x. CompAir can therefore compute the minimum across all sensor values x_i simply by computing the maximum over the bitwise complement of x_i, and then computing the bitwise complement of the computed result at the cluster head.

Going beyond minimum and maximum, CompAir can also compute percentiles, say, the median, using a binary search across the data. CompAir first computes the minimum and maximum as described above, as well as the total number of nodes as described in §4. It then computes the function $count(min < value < (min + max)/2)$ over the air, as described in §4. If this count is less than half the total number of nodes, it moves the interval to the right, *i.e.*, it computes the function $count((min + max)/2 < value < 3 * (min + max)/4)$. If not, it moves the interval to the left, *i.e.*, it computes $count((min + max)/4 < value < (min + max)/2)$, and so on, till it determines the boundary below which 50% of the data lies. Note that while we have described a simplified algorithm above, CompAir can achieve significantly higher performance since it computes the actual counts of sensors with values in each interval in each round. With prior knowledge of the distribution of the data, CompAir can therefore intelligently adapt the width of its search intervals, instead of simply halving the interval each time. For instance, consider the case when data is uniformly distributed between the minimum and maximum values. If the function $count(min < value < (min + max)/2)$ yields a value, say, $0.6N$, then CompAir can simply pick its next query to be $count(min < value < (0.5/0.6) * (min + max)/2)$, as the median is likely to be very close to this new value.

6. ROBUSTNESS OF COMPAIR TO NOISE

Noise is a critical factor in any wireless system. Thus, in this section we investigate how CompAir interacts with noisy channels, and the impact of noise on throughput gain.

6.1 Linear Functions

We consider the performance of CompAir in the computation of linear functions, *e.g.*, sum. In a noise-free scenario, sensors transmit their scaled measurements concurrently, and the cluster-head receives, in one shot, the sum as in Eq. 2. Such a simplified model however is likely to yield an erroneous sum in practice due to two types of noise:

- First, each received value incurs additive noise, which is a combination of the receiver's thermal noise and quantization error. As a result, the received sum after one transmission is likely incorrect, at least in its least significant bits. To increase robustness to this noise, CompAir repeats the transmission of the sum M_1 times and the cluster-head averages the received values.
- Second, since the sensor measurements in Eq. 2 have to be compensated by the channel before transmission, there is a contribution to noise from errors in the compensation factor. These errors are due to errors in the estimation of the channel $\hat{h}_i$ from each sensor to the cluster-head. To address these errors, the cluster-head broadcasts a request packet with M_2 channel estimation samples, where the M_2 samples are used by each sensor for averaging its channel estimate.

Since noise terms are independent, averaging reduces noise power by a factor equal to the number of averaged terms. However, how should the system pick the values of M_1 and M_2? And, how do these repetitions impact CompAir's throughput gain over a traditional system that performs the aggregation at a cluster-head?

We consider a simple model with a group of N sensors communicating with a cluster-head. We compare CompAir with a traditional transmission scheme that uses similar transmission power and bandwidth, and that achieves the same data resolution for the computed function. We define a sample as the value from an ADC operating at the Nyquist rate, *i.e.*, twice the wireless bandwidth of the signal.

Let the traditional scheme require a transmission of M_D samples from each sensor to the cluster-head. The total number of transmissions required by the traditional scheme to get data from all N sensors, therefore, is NM_D.

In contrast, in CompAir, The cluster-head transmits a request packet with M_2 channel estimation samples to the sensors. Each sensor $i, 1 \leq i \leq N$, estimates its channel to the cluster-head, $\hat{h}_i$, using these M_2 channel estimation samples. The sensors then jointly transmit their data to the cluster-head, with each sensor compensating by its channel estimate $\hat{h}_i$. They repeat this joint transmission M_1 times. The total number of samples transmitted by CompAir, *i.e.*, its total overhead, is $M_A = M_1 + M_2$. We can therefore compute the throughput gain provided by CompAir as $\alpha = \frac{NM_D}{M_1+M_2}$.

But how large should M_1 and M_2 be? The values of M_1 and M_2 are chosen based on the desired bit resolution, b, of the function. Specifically, in order to get a resolution of b bits, the SNR of the sum has to be $6.02b + 1.76$dB (see [24] for derivation). Note that the SNR in computing the sum is different from the SNR of the channel due to repetition and averaging. Specifically, the SNR in computing the sum depends on the averaging parameters M_1 and M_2, which are chosen to reduce the estimation noise and increase the SNR. Let us refer to the SNR of the sum as the effective SNR, which we will compute below.

Let the average transmit power of each sensor be P, and the average additive Gaussian noise at the receiver σ. Then, the channel SNR experienced by a traditional system is, by definition, $SNR = \frac{P}{\sigma^2}$.

In CompAir, however, sensors transmit jointly. Further their transmitted samples add coherently since they are scaled by the magnitude and phase of the channels. This translate into an SNR gain for the joint transmission. Specifically, in each joint transmission, each sensor transmits with an average power of P, *i.e.*, the average magnitude of the transmitted signal from each sensor is $\sqrt{P}$. Since the sensors align their transmitted signals coherently, the magnitude of the received signal at the cluster-head is $N\sqrt{P}$, and consequently, the power of the received signal is N^2P.[1]

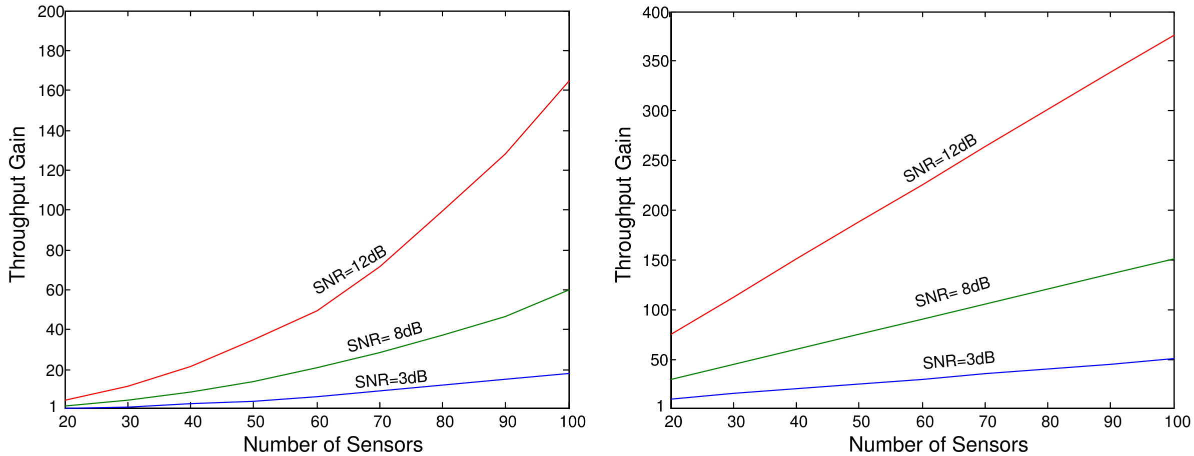
When we average M_1 such joint transmissions, the average power combines coherently whereas noise combines incoherently, because it is independent across the different receptions. As a result, the average received power after averaging M_1 joint transmission is N^2P and the average received noise power is $\frac{\sigma^2}{M_1}$.

There is a further noise contribution from errors in estimation of the channel to the cluster-head. Since the channel estimate is averaged at each sensor across M_2 samples, this reduces the expected noise in the channel estimate at each sensor to $\frac{\sigma^2}{M_2}$. Further, this noise adds up (incoherently as before) across all the sensors in a joint transmission, producing a total contribution of channel estimation noise equal to $\frac{N\sigma^2}{M_2}$.

Combining these two, we therefore get that the total signal power is N^2P and the total noise power is $\frac{\sigma^2}{M_1} + \frac{N\sigma^2}{M_2}$. The effective SNR of the system (i.e., the SNR of the computed sum), therefore is the ratio of these two terms, *i.e.*,

$$SNR_{eff} = \frac{N^2P}{\frac{\sigma^2}{M_1} + \frac{N\sigma^2}{M_2}}$$

[1] Note that, similar to beamforming, the fact that N signals, each with power P, produce a total power of N^2P at the cluster-head does not violate conservation of power. This is because, the average power across all points in space still remains the same, and aligning the transmitted signals only reshapes the power profile to maximize the power at the cluster-head as if the N sensors were an N-antenna MIMO transmitter that beamforms its signal to the cluster-head.



(a) Computing the *sum* function

(b) Computing the max function

Figure 1— CompAir Throughput Gain: Figure shows that CompAir provides a throughput gain relative to Zigbee for computing functions across a wide range of channel SNR and different number of sensors.

Since the original channel SNR is $\frac{P}{\sigma^2}$, we can therefore write

$$SNR_{eff} = \frac{N^2 \times SNR}{\frac{1}{M_1} + \frac{N}{M_2}}$$

As described earlier, in order to get the desired b bits of resolution, the SNR requirement of the system can be written as $dB(SNR_{eff}) = 6.02b + 1.76$, where the function $dB(x) = 10\log_{10}(x)$. In order to maximize the throughput gain, therefore, the CompAir cluster-head minimizes $M_1 + M_2$ subject to the SNR requirement. This convex optimization problem has a closed form solution, and the cluster-head can therefore simply determine the optimal values of M_1 and M_2.

Specifically, the optimal values are:

$$M_1 = \frac{1+\sqrt{N}}{lN} \text{ and } M_2 = \sqrt{N}M_1$$

where l is defined by the equation $dB(l) = dB(N \times SNR) - (6.02b + 1.76)$, *i.e.*,

$$l = N \times SNR \times 10^{\frac{-(6.02b+1.76)}{10}}$$

Based on these optimal values, we can compute the throughput gain of CompAir as

$$\alpha = \frac{N^3}{(\sqrt{N}+1)^2} \times SNR \times M_D \times 10^{\frac{-(6.02b+1.76)}{10}}$$

The throughput gain for linear-functions therefore scales linearly with channel SNR (*i.e.*, exponentially with SNR expressed in dB), and approximately quadratically with the number of sensors.

6.2 Non-linear Functions

As described earlier, non-linear function computation proceeds in rounds, with CompAir computing an “OR” function of the transmitted bits in each round. The “OR” function is inherently robust because it only requires differentiating the presence of power on the medium from the absence of power (apart from receiver noise), and its performance is a lower bound on any receivers which need to detect power on the medium to begin decoding a packet. We defer the analytical modeling of the performance of CompAir for non-linear functions for a full version of the paper, and present simulation results in 7.

7. Evaluation

We present simulation results that show CompAir’s scaling behavior for a large number of sensors. We also present initial implementation results using a USRP testbed to demonstrate that the design can be built in real radios and its behavior matches the analysis.

7.1 CompAir’s Throughput Scaling

Method: The chief promise of CompAir is its ability to provide increasing throughput gains with increasing density of sensors. In order to test CompAir’s performance in the many sensor regime, we build a sensor network simulation framework. We deploy N sensors in a $10m \times 10m$ grid (on the order of ZigBee transmission range), with a single cluster-head for the grid. We increase the sensor density, by varying N from 20 to 100. For each N, we vary the average SNR from the sensors to the cluster-head. We set the desired bit resolution to 8 bits and use the average SNR to compute the parameters M_1 and M_2, as described in §6. The receiver noise is generated using an additive white Gaussian model, where the noise variance is scaled according to the average SNR in the run. We compare against a traditional system that aggregates the measurements at the cluster-head by having each sensor transmit its 8-bit measurement to the cluster head using ZigBee. We do not simulate physical-layer headers and ZigBee medium contention overhead. Both systems use 2 MHz bandwidth as typical in ZigBee. The ZigBee system uses its typical transmission rate of 250 kbps [8]. CompAir transmits in accordance with the description in §6 and §4 for linear functions and the description in §5 for non-linear functions. The ADC sampling rate for both systems is assumed to be twice the bandwidth in accordance with the Nyquist criterion.

Results: Sum: Fig. 1(a) depicts the throughput gain of CompAir relative to ZigBee for computing the *sum* function while varying the number of sensors and the average SNR. We see that CompAir provides a throughput gain across the entire range of SNRs. The throughput gain scales roughly quadratically with the number of sensors. For example, for a dense network (*i.e.*, 100 sensors with an average of 12 dB), CompAir's throughput is 165× that of the traditional ZigBee system when computing a linear function. As expected the gain is small for low density networks where the number of sensors per cluster-head is about 20. Hence, CompAir is particularly useful for dense large sensor networks.

Max: Fig. 1(b) plots the throughput gain for computing the max function for a bit resolution of 8. The throughput gain scales linearly with the number of sensors. Note that CompAir naturally provides a linear gain with the number of sensors because the traditional system requires a linear increase in the number of unicast transmissions as the number of sensors increases, whereas the number of transmissions in CompAir stays fixed. Note that linear functions provide an additional gain since all sensors participate in all joint transmissions providing a corresponding increase in power and hence SNR. In contrast, with *max*, only the sensors with a bit value of 1 in the corresponding position participate in a round, and the number of participating sensors decreases as the number of rounds increases.

7.2 Preliminary Implementation Results

We show preliminary implementation results that demonstrate the practicality of building CompAir in real radios.

Method: We implement a prototype of CompAir using USRPs. The USRPs share a reference clock using AirShare [1]. The shared reference clock ensures that all transmissions are coherent. The implementation of AirShare follows the description in [1] and has been verified with the authors of that paper.

We place the USRPs at random locations in an indoor testbed, corresponding to locations of sensors and a cluster-head. The sensors each uniformly pick random values between 0 and 1. We configure the CompAir system to compute the *sum* function across these values. Specifically, the cluster-head initiates joint transmission with its channel estimation packet. The sensors then compensate for the estimated channel and transmit their scaled values coherently. The cluster-head computes the received sum. The sensors keep repeating their joint transmissions, and the cluster-head iteratively averages the received values to update its computed sum. We compare this computed sum from CompAir to the actual true value of the sum based on the original value at each sensor.

Overall the experiment uses 6 USRPs as sensors and one

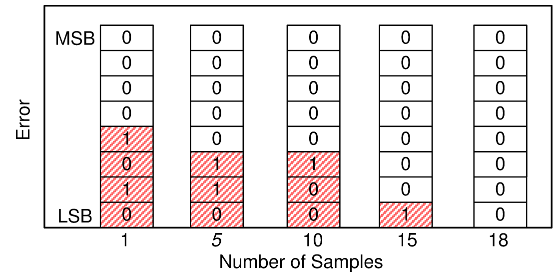


Figure 2—Experimental Results for 24 Sensors: This experiment shows that the noise affects the lower order bits of the sum value. The error goes down by averaging more samples.

USRP as a cluster-head. However, since CompAir is particularly designed for a large sensor network, we emulate the effect of many sensors by repeating the experiment with a batch of 6 USRPs and combining their received signals in post-processing. For example, to create a scenario with 24 sensors we repeat the joint transmission of 6 sensors 4 times, where each of these repetitions corresponds to different locations of the USRPs. We then add the four received transmissions to create one joint transmission of 24 sensors. Note this is a conservative estimate since we are adding up the noise added by receiver in four experiments.

Result: Fig. 2 shows the results of computing an 8-bit sum over the measurements of 24 sensors. The figure shows the the bit representation of the error from MSB to LSB, where the error refers to the difference between the sum computed at the cluster-head after running CompAir, and the actual sum computed over the original sensor values. As can be seen in the figure, the errors are concentrated towards the LSB with the most significant 4 of the 8 bits accurate even after 1 transmission. This corresponds to our intuition that noise tends to affect the lower order bits of the *sum* value. Further, the figure shows that the error goes down as we average more samples and eventually goes down to 0. This shows that averaging reduces the impact of noise, and that our CompAir implementation effectively delivers coherent transmission across all sensors.

In this experiment, we need less than 18 samples to represent the sum of value from 24 sensors with 8-bits of resolution. In this experiment, the average SNR per sensor (*i.e.*, the SNR when a single sensor transmits alone) was about 3 to 4 dB.[2] Thus, if each sensor were to transmit alone, the system would have to use BPSK and each sensor out of the 24 sensors would need at least 8 transmissions (ignoring coding) for a total of 192 transmissions. In contrast, CompAir computes the sum in less than 18 transmissions.

[2] The reason that the first transmission can deliver 4 correct bits is because the joint transmission has a higher power/SNR than individual transmission due to the sensors transmitting jointly and coherently.

8. CONCLUSION

This paper proposes a novel over-the-air function computation mechanism to compute a wide variety of popular linear and non-linear functions over wireless sensor measurements. The mechanism leverages simultaneous coherent transmission from multiple sensors and modifies the sensor transmissions so that their collision on the wireless channel produces the desired function value.

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