UNDERSTANDING THE COMPUTATIONAL DEMANDS UNDERLYING VISUAL REASONING

A PREPRINT

Mohit Vaishnav^{1,2}, Remi Cadene², Andrea Alamia³, Drew Linsley², Rufin VanRullen^{1,3}, and Thomas Serre^{1,2}

¹Artificial and Natural Intelligence Toulouse Institute, Université de Toulouse, France ²Carney Institute for Brain Science, Dpt. of Cognitive Linguistic & Psychological Sciences, Brown University, Providence, RI 02912

³Centre de Recherche Cerveau & Cognition (CerCo), CNRS, Universite de Toulouse, 31052 Toulouse, France

Keywords: visual relations, deep learning, convolutional neural networks, transformer networks, spatial attention, feature-based attention

Abstract

Visual understanding requires comprehending complex visual relations between objects within a scene. Here, we seek to characterize the computational demands for abstract visual reasoning. We do this by systematically assessing the ability of modern deep convolutional neural networks (CNNs) to learn to solve the "Synthetic Visual Reasoning Test" (SVRT) challenge, a collection of twenty-three visual reasoning problems. Our analysis leads to a novel taxonomy of visual reasoning tasks, which can be primarily explained by both the type of relations (same-different vs. spatial-relation judgments) and the number of relations used to compose the underlying rules. Prior cognitive neuroscience work suggests that attention plays a key role in human's visual reasoning ability. To test this, we extended the CNNs with spatial and feature-based attention mechanisms. In a second series of experiments, we evaluated the ability of these attention networks to learn to solve the SVRT challenge and found the resulting architectures to be much more efficient at solving the hardest of these visual reasoning tasks. Most importantly, the corresponding improvements on individual tasks partially explained the taxonomy. Overall, this work advances our understanding of visual reasoning and yields testable Neuroscience predictions regarding the need for feature-based vs. spatial attention in visual reasoning.

1 Introduction

Humans can effortlessly provide rich and detailed descriptions of briefly presented real-life photographs (Fei-Fei et al., 2007) – vastly outperforming the best current computer vision systems (Geman et al., 2015; Kreiman and Serre, 2020). Most previous human studies have typically focused on understanding the neural computations underlying the judgment of individual relations between objects, such as their spatial relations (e.g., Logan (1994a)) or whether they are the same or different (up to a transformation, e.g., Shepard and Metzler (1971)). Prior Cognitive Neuroscience studies have shown that different visual reasoning problems have different attentional and working memory demands (Logan, 1994b; Moore et al., 1994; Rosielle et al., 2002; Holcombe et al., 2011; Van Der Ham et al., 2012; Kroger et al., 2002; Golde et al., 2010; Clevenger and Hummel, 2014; Brady and Alvarez, 2015). However, to date, relatively little is known about the underlying computational basis of same different discrimination and visual reasoning in general (see Ricci et al. (2021) for a recent review).

One benchmark that has been designed to probe abstract visual relational capabilities in humans and machines is the *Synthetic Visual Reasoning Test* (SVRT) (Fleuret et al., 2011). The dataset consists of twenty-three hand-designed binary classification problems with stimuli consisting of simple closed-contours shapes and aimed at testing the recognition of abstract concepts. Observers are never explicitly given the underlying classification rule and they must

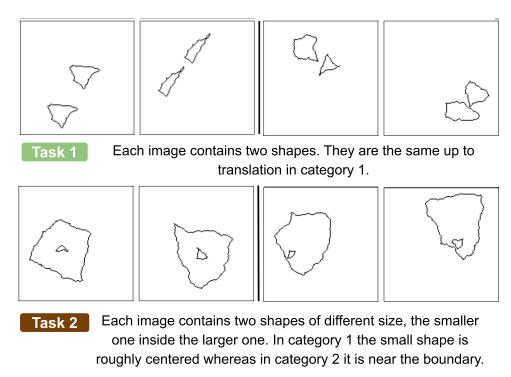


Figure 1: Two SVRT sample tasks from a set of twenty three in total. For each task, the leftmost and rightmost two examples illustrate the two categories to be classified. Representative samples for the complete set of twenty-three tasks

can be found in Figure S2 and S3.

learn it from the presentation of positive and negative training examples. Examples from two representative tasks are depicted in Figure 1: observers must learn to recognize whether two shapes are the same or different (Task 1), or whether or not the smaller of the two shapes is near the boundary (Task 2). Additional relations represented in the challenge include "inside", "in between", "forming a square", "aligned in a row" or "finding symmetry" (see Figures S2 and S3 for examples).

Most SVRT tasks are rapidly learned by human observers within twenty or fewer training examples (Fleuret et al., 2011) (see Table 2; reproduced from the original study). Modern deep neural network models require several orders of magnitude more training samples for some of the more challenging tasks (Ellis et al., 2015; Stabinger et al., 2016a; Kim et al., 2018; Messina et al., 2021; Stabinger et al., 2021, 2016b) (see Ricci et al. (2021) for review; see also (Funke et al., 2021) for an alternative perspective).

Prior work has started to reveal an initial dichotomy of SVRT tasks. For instance, it has been found that tasks that involve spatial-relation (SR) judgments can be learned much more easily by deep convolutional neural networks (CNNs) than tasks that involve same-different (SD) judgments (Stabinger et al., 2016b; Kim et al., 2018; Yihe et al., 2019). This is not to say that CNNs cannot learn SD tasks (obviously they will given sufficient training samples given that they are universal approximators) – only that CNNs require far more training examples and/or more processing layers to successfully learn to solve SD vs. SR task. The implication is that CNNs would appear to need additional computations beyond the filtering, non-linear rectification, and pooling that they rely on to learn SD tasks as efficiently as SR tasks. Indeed, recent human electrophysiology work (Alamia et al., 2021) has shown that SD tasks recruit cortical mechanisms associated with attention and working memory processes to a greater extent than SR tasks but state-of-the-art CNNs lack such mechanisms. However, beyond this basic dichotomy (SR vs. SD), little is known about the neural computations necessary to solve SVRT tasks as efficiently as human observers.

Here, we aim to further our understanding of the neural computations required for visual reasoning in two sets of experiments. In our first set of experiments, we extend prior studies on the learnability of individual SVRT tasks by feedforward neural networks using a popular class of deep neural networks known as deep residual networks ("ResNets") (He et al., 2016). We systematically analyze the ability of ResNet architectures to learn all twenty-three SVRT tasks as a function of their processing depth (or number of layers – as a proxy for their expressiveness) and the

number of samples used for training (to characterize their sample efficiency for that particular task). Through these experiments, we found that most of the variance in the space of SVRT tasks could be accounted for by two principal components, which reflected both the nature (same-different vs. spatial-relation judgments) and the number of relations used to compose the underlying rules.

Consistent with Neuroscience studies on visual reasoning (Egly et al., 1994; Roelfsema et al., 1998), prior work by Kim et al. (2018) has shown that combining CNNs with an oracle model of attention and feature binding (i.e., preprocessing images so that they are explicitly and readily organized into discrete object channels) renders SD tasks as easy to learn by CNNs as SR tasks. Here, we build on this work and describe CNN extensions to incorporate spatial and feature-based attention. In a second set of experiments, we show that these attention networks learn difficult SVRT tasks with fewer training examples than their pre-attentive (CNN) counterparts. We also find that spatial and feature-based attention improved model performance on different tasks. A finer analysis further revealed a close correspondence between the relative improvements obtained following the addition of these two forms of attention and the taxonomy identified in experiment 1.

This previous set of experiments leaves open the question as to how these attention mechanisms contribute to learning these visual reasoning problems. One could imagine two possible contributions of attention mechanisms in helping these networks to learn better visual representations for the task and/or to learn the abstract rule more efficiently. To control for the learnability of visual features, we contrasted two scenarios, one where the models were trained from scratch such that they have to learn both SVRT features and the rule and one where they were pre-trained on SVRT such that they only have to learn the rule. We found the task benefits of pre-training to learn SVRT features to be correlated with the benefits of spatial attention – suggesting that spatial attention helps discover the rule more so than learning good visual representations.

2 Experiment 1: ResNets

2.1 Systematic analysis of SVRT tasks' learnability

All experiments were carried out with the *Synthetic Visual Reasoning Test* (SVRT) dataset using code provided by the authors to generate images with dimension 128×128 pixels (see Fleuret et al. (2011) for details). In this experiment we want to show the learnability of these tasks using ResNets. We trained 18-, 50-, and 152-layer ResNets separately on each of the SVRT's twenty-three tasks. In addition, we also trained networks using different number of training examples. Estimates of the changes in accuracy as a function of the number of samples for a fixed depth ("sample complexity") and changes in accuracy as a function of depth for a fixed number of samples (i.e., how expressive the network has to be) provide complementary characterization of the learnability of these tasks. Our datasets contained .5k, 1k, 5k, 10k, 15k, and 120k class-balanced samples. We also generated two unique sets of 40k positive and negative samples for each task: one was used as a validation set to select a stopping criterion for training the networks (if validation accuracy reaches 100%) and one was used as a test set to report model accuracy. In addition, we used three independent random initializations of the training weights for each configuration of architecture/task and selected the best model using the validation set. Models were trained for 100 epochs using the Adam optimizer (Kingma and Ba, 2014) with a training schedule (we used an initial learning rate of 1e-3 and changing it to 1e-4 from 70^{th} epoch on-wards). As a control, because these tasks are quite different from each other, we also tested two additional initial learning rates (1e-4, 1e-5).

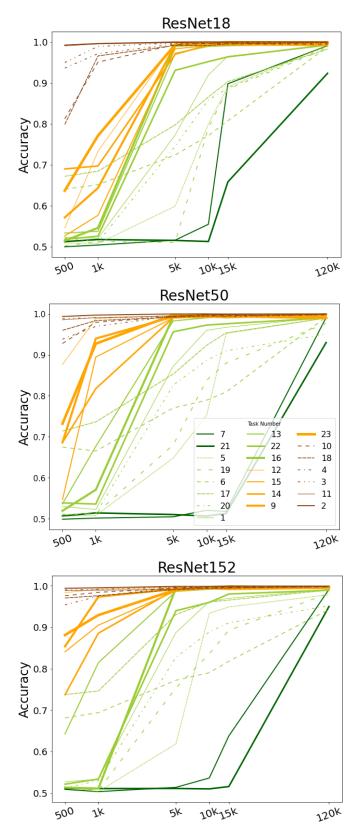


Figure 2: Test accuracy for each of the twenty-three SVRT tasks as a function of the number of training samples for ResNets with depth 18, 50 and 152, resp. The color scheme reflects the identified taxonomy of SVRT tasks (see Figure 3 and text for details).

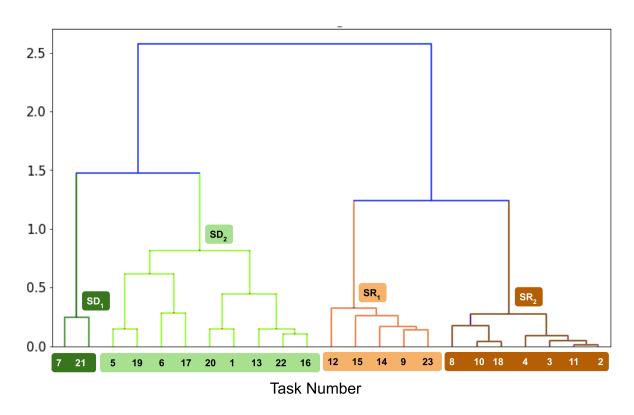


Figure 3: Dendrogram derived from an N-dim hierarchical clustering analysis on the test accuracy of N=15 ResNets[18/50/152] trained to solve each task over a range of training set sizes.

Figure 2 shows the test accuracy of the three ResNets with depth 18, 50, and 152 for each SVRT task as a function of the number of samples used for training. Consistent with prior work (Kim et al., 2018; Stabinger et al., 2016b; Yihe et al., 2019), we found that some SVRT tasks are much easier to learn than others for ResNets. For instance, a ResNet50 needs only 500 examples to perform well on tasks 2, 3, 4, 8, 10, 11, 18 but the same network needs 120k samples to perform well on task 21 (see Figures S2 and S3 for examples of these tasks). Similarly, with 500 training examples, task 2, 3, 4 & 11 can be learned well with only 18 processing stages while task 9, 12, 15 & 23 require as many as 152 processing stages. A key assumption of our work is that these differences in training set sizes and depth requirements between different SVRT tasks reflect different computational strategies that need to be discovered by the neural networks during training for different tasks. Our next goal is to better understand what these computational strategies are.

2.2 An SVRT taxonomy

To further characterize the space of SVRT tasks, we performed a multi-variate clustering analysis. For each individual task, we created an N-dimensional vector by concatenating the test accuracy of all ResNet architectures (N=3 depths \times 5 training set sizes = 15) as discussed above as a signature of each task's computational requirements. We then performed a hierarchical clustering analysis using agglomerative methods; the resulting dendrogram is shown in Figure 3.

Our clustering analysis reveals a novel taxonomy of visual reasoning tasks: At the coarsest level, it recapitulates the dichotomy between *same-different* (SD; green branches) and *spatial-relation* (SR; red branches) categorization tasks originally identified by Kim et al. (2018) using shallow CNNs and univariate analyses. Interestingly, two of the tasks which were classified as SR by Kim et al. (2018) (tasks 6 & 17) were assigned to the SD cluster in our analysis. We examined the descriptions of these two tasks as given in Fleuret et al. (2011) (see also Figures S2 and S3) and found that these two tasks not only involve an SR component but also an SD component as they require the identification of two same shapes in an image and then apprehending the distance between those. Task 6 involves two pairs of identical shapes with one category having same distance in-between two identical shapes vs. not in the other. Similarly in task 17, three of the four shapes are identical and their distance with the non identical one is same in one category vs. different in the other. Thus, the assignment of tasks into SR vs. SD identified in our experiment seems somewhat cleaner than

that identified by Kim et al. (2018). This could be due to our use of ResNets (as opposed to vanilla CNNs), deeper networks, and a greater variety of training set sizes (including much smaller training set sizes than those used by Kim et al. (2018)).

Beyond this main dichotomy, our multi-variate analysis revealed a finer organization. The SR cluster could be further subdivided into two sub-clusters. A dark-brown-coloured branch involves relatively simple and basic relation rules such as shapes making close contact (3, 11), or being close to one another (2), one shape being inside the other (4) or whether the shapes are arranged to form a symmetric pattern (8, 10, 18). On the contrary, tasks that fall in the light-brown-coloured branch involve the composition of more than two rules such as comparing the size of multiple shapes to identify a subgroup before identifying the relationship between the members of the sub-groups. This includes tasks such as finding a larger object in between two smaller ones (9) or three shapes of which two are small and one large with two smaller (identification of large and small object) ones either inside or outside in one category vs. one inside and the other outside in the second (23), or two small shapes equally close to a bigger one (12), etc. These tasks also tend to be comparatively harder to learn by the neural networks requiring greater depth or number of training samples. For instance, contrast tasks 9, 12, 15, 23 with tasks 2, 4, 11 in training regimes with fewer training samples or tasks 12, 15, 9, 23 as they require more deeper network (Figure 2).

We found that task 15 gets assigned to this latter sub-cluster because the task requires finding four shapes in an image that are identical vs. not. One would expect this task to fall in the SD cluster but we speculate that the deep networks are actually able to leverage a shortcut (Geirhos et al., 2020) by classifying the overall pattern as symmetric/square (when the four shapes are identical) vs. trapezoid (when the four shapes are different; see Figure S3) – effectively turning an SD task into an SR task. Further work should try to address this potential bias in the SVRT task 15 to test our assumption.

Similarly, our analysis reveals a further subdivision of the SD cluster. All these tasks require recognizing shapes which are identical to at least one of the other shapes in the image. The first sub-cluster belongs to tasks that require the identification of simple rules such as detecting whether two shapes are identical (even if it is along the perpendicular bisector) (tasks I, 20; see Figure S2), or identifying if all the shapes are same (I6, I6, I6) or finding if two pairs of identical shapes are translationally symmetrical (I6). Another set of tasks within this sub-cluster includes tasks that are defined by rules that include the composition of additional relational judgments. Sample tasks include identifying pairs/triplets of identical shapes and measuring the distance with the rest (I6, I7), or finding if identical shapes occur in pairs (I6), or detecting if one of the shapes is the scaled version of the other one (I6). Finally, the second sub-cluster shown in dark-green color involves two tasks that require an understanding of shape transformations including knowing whether one of the shapes is the scaled, translated or rotated version of the other one (I6) or complex relations such as counting as in judging whether an image contains two pairs of three identical shapes or three pairs of two identical shapes in an image (I7).

To summarize this first set of experiments, we have systematically evaluated the ability of ResNets spanning multiple depths to solve each of the twenty-three SVRT tasks for different training set sizes. This allowed us to represent SVRT tasks as N-dimensional vectors and we used clustering methods to better understand the resulting SVRT space. Beyond a main SD-SR dichotomy, our analysis revealed additional sub-clusters which could be explained by the number of rules (one single rule vs. two rules) that were composed together to define the task. Increasing the number of rules seems to make the task harder to learn for the ResNet. Interestingly, we also find an organization of the space by task complexity such that easier SR and SD sub-clusters fall close to each other with the harder SR and SD sub-clusters falling farther away. We next study the role of attention mechanisms to help solve those tasks that appear more challenging to be learned by CNNs.

3 Experiment 2: Attention networks

Attention processes in neuroscience can be broadly divided into *spatial* (e.g., attending to a particular image location) vs. *feature-based* (e.g., attending to a particular shape or color) processes (Desimone and Duncan, 1995). The importance of attention has also been recently realized by the computer vision community with a number of attention modules proposed to extend CNNs – including spatial (e.g., Sharma et al. (2015); Chen et al. (2015); Yang et al. (2016); Xu and Saenko (2015); Ren and Zemel (2016)), feature-based (e.g., Stollenga et al. (2014); Chen et al. (2017); Hu et al. (2018)) and hybrid (e.g., Linsley et al. (2018a); Woo et al. (2018)) approaches. Here, we adapt the increasingly popular transformer architecture to implement both forms of attention. These networks originally developed for natural language processing (Vaswani et al., 2017) are now pushing the state of the art in computer vision (Zhu et al., 2020; Carion et al., 2020a; Dosovitskiy et al., 2020b). Here, the spatial dimension of the output of the CNN layers is flattened before passing to our transformer module to get the spatial attention vs. we transpose this spatial dimension with

feature dimension before passing it to the transformer module to get feature-based attention. We refer the reader to the supplementary section S1 for details on our implementations.

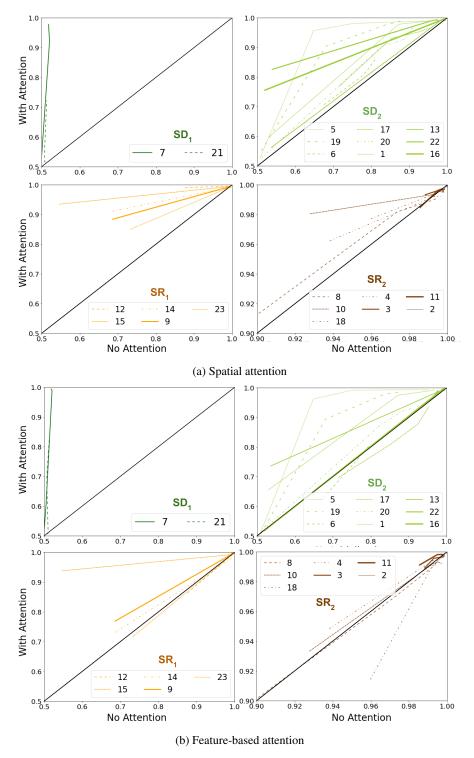


Figure 4: Test accuracies for a baseline ResNet50 vs. the same architecture endowed with the two forms of attention for each of the twenty-three SVRT tasks when varying the number of training examples. Also note that a different axis scale is used for SR_2 to improve visibility.



Figure 5: Test accuracies obtained using a vanilla ResNet50 with feature-based attention and spatial attention modules for 10k training samples.

We first compared the accuracy of two types of attention network (i.e., a ResNet50 endowed with a spatial attention module, and a ResNet50 endowed with a feature-based attention module) with that of a vanilla ResNet50. Shown in Figure 4 are the test accuracies obtained for each SVRT task with a ResNet50 endowed with spatial attention and feature-based attention plotted against the corresponding accuracies of a vanilla ResNet50 (i.e., without attention). Each curve corresponds to a different task derived by training these networks for different number of training examples (not shown; higher accuracies correspond to training with larger number of samples). The vanilla ResNet50 provides a baseline to gauge any improvement associated with the addition of these feature-based and spatial attention mechanisms. Tasks are grouped into sub-plots corresponding to the sub-clusters identified in Experiment 1 (two main clusters for SD vs. SR and their associated sub-clusters). To recall, the two sub-clusters from SD are represented as SD_1 and SD_2 which consist of tasks (7, 21), (5, 19, 6, 17, 20, 1, 13, 22, 16) respectively. We further labeled the SR sub-clusters as SR_1 and SR_2 , corresponding to the following task numbers: (12, 15, 14, 9, 23) and (8, 10, 18, 4, 3, 11, 2), respectively.

For all tasks, spatial attention consistently improves the network's accuracy for any number of training examples. The improvement in accuracy is particularly noticeable for the SD_1 cluster. We have already shown that tasks involved in this sub-cluster are associated with a composition of two rules and all the tested networks struggled to learn the tasks without attention. Adding an attention module to the ResNet50 network helps it learn these tasks more efficiently. The improvement is also robust to a lesser extent for SD_2 and SR_1 and the improvement is marginal for SR_2 because the vanilla architecture can already learn the tasks relatively well (note the use of a different axis for the corresponding plot).

We find that feature-based attention leads to the highest improvements for SD_1 especially in intermediate training regimes (Figure 5) while spatial attention leads to the highest improvements for SD_2 and SR_1 especially in lower training regimes (corresponding on the plot to lower test accuracy regimes for the baselines ResNet50). SR_2 tasks are learned well without any attention mechanisms with as little as 500 training examples. Overall, these results validate our hypothesis that different sub-clusters correspond to tasks that exhibit different computational demands.

We further performed a principal component analysis (see Figure 6) from the feature vector (N=15) derived in Experiment 1 – consisting of the test accuracies of the twenty-three SVRT tasks for the baseline ResNet models of varying depths (18, 50, 152 layers) and five dataset sizes (.5k, 1k, 5k, 10k & 15k). We selected the top two principal components for display which together account for $\sim 93\%$ of the variance. With the exception of task 6, the four sub-clusters discovered in Experiment 1 are already visible when only considering the first two principal components (see dotted red lines). Interestingly, the first principal component alone is sufficient to recapitulate the SD vs. SR

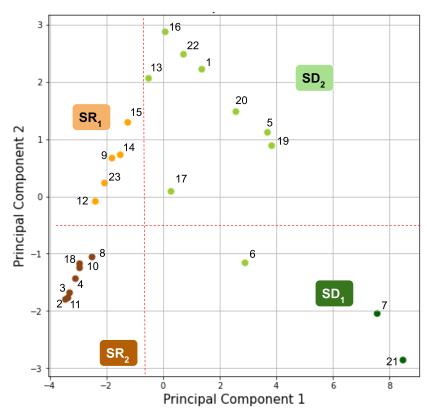


Figure 6: Principal component analysis of the twenty-three tasks using the 15-dimensional feature vectors derived from Experiment 1 representing the test accuracy obtained for each task for different dataset sizes and ResNets of varying depths (18, 50 & 152). Dotted red line represents 4 different bins in which these tasks can be clustered.

dichotomy while the second principal component seems to capture the tasks attentional demands for spatial attention (recall that SD_1 vs. SD_2 benefit most from *feature-based* vs *spatial* attention; see Figure 4).

To better understand how the ResNet-derived taxonomy found in Experiment 1 can be explained by the need for spatial and feature-based attention, we performed an additional analysis whereby we considered the test accuracy of the spatial and feature-based attention architectures for different dataset sizes (.5k, 1k, 5k, 10k, 15k). For each dataset size, we calculated the ratio of the test accuracy of two attention processes individually with that of the vanilla ResNet50 for each task and training condition. A ratio of 1 means no improvement with the corresponding attention process for that particular task and training set size. A ratio greater/smaller than 1 corresponds to an increment/decrement in accuracy for that task with the addition of the corresponding attention process. This gives us five ratios per task and attention process corresponding to each dataset size. We then linearly fitted these points and calculated the corresponding slope. This slope characterizes the relative benefit of attention for that particular task as the number of training examples available increases. A negative slope means that the benefit of attention is most evident in lower training regimes; conversely a positive slope means that the benefit of attention is most evident in higher training regimes. Two representative examples of positive and negative slopes are shown in Figure 7 for tasks 7 and 22. Repeating this procedure for all twenty-three tasks and for both spatial and feature-based attention gives us two 23-dimensional vectors.

These slope vectors can be found in Figure S4 and S5 for spatial attention and in Figure S6 and S7 for feature-based attention. In general we can see that for any form of attention there is an increasing trend in slope as we go from SR to SD. This suggests that tasks belonging to SR category requires no attention or minimal attention whereas SD_1 requires some form of attention, preferably feature-based attention over spatial attention. In sub-cluster SR_1 , we find a surprisingly relatively high slope value for task 15 which is consistent with our results found in Experiment 1 and consistent with our hypothesis that the networks are capable to use a shortcut to learn this SD task and treat it as an SR task instead.

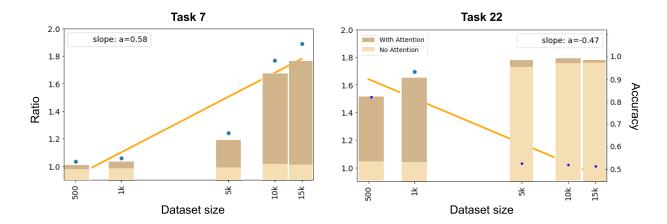


Figure 7: Understanding in what training regime attention helps. We first compute the ratio of the test accuracy of each of the two attention processes with the test accuracy of a baseline ResNet50 for each task and training condition. As an example, these ratios are plotted for different training set sizes for tasks 7 and 22. When the ratio is greater than 1 it shows that attention helps vs. hurts when lower than 1. This gives us five ratios per task and attention process corresponding to each dataset size. We performed a linear fitting procedure for these points and calculated the corresponding slope. This slope characterizes the relative benefits of attention for that particular task as the number of training examples available increases. If the benefit of attention is most evident in lower training regimes, one would expect a relatively small slope and if the benefit of attention is most evident in higher training regimes, one would expect a large slope.

Table 1: Pearson coefficient (r) and corresponding p values obtained by correlating the slope vectors of the spatial attention and the feature-based attention modules with the two principal components of Figure 6. See text for details.

	Spa	atial	Feature					
	r	p	r	p				
PC_1	0.466	0.0249	0.649	0.0008				
PC_2	-0.652	0.0007	-0.491	0.0174				

We then correlated these vectors with the principal components (PC_1, PC_2) from Figure 6. The corresponding Pearson coefficient r and p values are given in Table 1. Interestingly, we find a dissociation between the two forms of attention with feature-based attention correlating most with the first principal component and spatial attention with the second principal component. Note that the sign of the projections along the principal components is arbitrary and hence the sign of the correlation should not be interpreted beyond just the fact that there is correlation.

To summarize our results from Experiment 2, we have found that the task clusters derived from ResNets' test accuracies computed over a range of depth and training set sizes can be explained in terms of attentional demands. ResNets are universal approximators, and as expected, with sufficient amount of training data, any sufficiently deep ResNet learns all twenty-three problems. Here, we have shown that endowing these networks with attentional mechanisms helps them learn some of the most challenging problems with far fewer training examples. We also found that the relative improvements obtained over standard ResNets with feature-based and spatial attention are consistent with the taxonomy of visual reasoning tasks found in Experiment 1. More generally, our analysis shows how the relative need for feature vs. spatial attention seems to account for a large fraction of the variance in the space of SVRT tasks defined in Experiment 1 according to their learnability by ResNets.

4 Experiment 3: Feature vs. rule learning

In this section, we contrast learning of SVRT tasks from the perspective of two network models. One model was trained to solve the problems from scratch as done in Experiment 1 and 2 (i.e., using randomly initialized weights) such that both the visual features (i.e., by convolutional layers) and the classification rule (i.e., by the final linear layer in ResNets) were learned during training. The other model was pre-trained using an auxiliary task in order to learn visual features that are already adapted to the SVRT stimuli such that only the classification rule needs to be learned during

the actual training as opposed to learning a mixture of visual features and rule for the network trained from scratch. For pre-training, we used an image dataset consisting of 5,000 samples (class balanced) for each of the twenty-three tasks $(5,000 \times 23 = 115,000 \text{ samples in total})$. To make sure the networks did not learn any of the SVRT abstract rules, we shuffled images and binary class labels across all twenty-three problems while pre-training the network. For the actual training, we froze the weights of the convolutional layers and fine-tuned only the classification layers to learn the actual rule for each of the SVRT problems.

As a control, we trained a third architecture that was pre-trained on ImageNet (a popular computer vision dataset containing natural object categories; Deng et al., 2009). This provides a good benchmark to demonstrate that our SVRT-pre-training approach does indeed learn meaningful representations. This approach is similar in spirit to that of (Goyal and Bengio, 2021) who explored the link between inductive bias vs. learnability of a task and generalization.

Figure 8 shows a comparison between the different architectures in terms of their test accuracies according to the sub-clusters discovered in Experiment 1. These results first confirm that the SVRT pre-training approach works because it consistently outperforms pre-training on ImageNet (Figure S8) or training from scratch. Interestingly, for the SR_2 sub-cluster, we found that the benefits of pre-training on SVRT goes down very quickly as the number of training examples grows. We interpret these results as reflecting the fact that generic visual features are sufficient for the task and that the rule can be learned very quickly (somewhere around 500 and 5,000 samples). For SR_1 sub-cluster, the benefits of starting from features learned from SVRT is somewhat more significant in low training regimes but these advantages quickly vanish as more training examples are available (the task is learned by all architectures within 5,000 training samples).

For SD_1 while there appears to be a significant advantage of pre-training on SVRT over ImageNet pre-training and training from scratch, the tasks never appear to be fully learned by any of the networks even with 15,000 training examples. This demonstrates the challenge of learning the rules associated with this sub-cluster beyond simply learning good visual representations. Finally, our results also show that the performance gap across all the architectures for SD_2 vs. SD_1 increases rapidly with more training examples – demonstrating the fact that the abstract rule for SD_2 tasks are more rapidly learned than for SD_1 .

At last, we carried out a similar analysis with the pre-trained network as done in Experiment 2: We built test accuracy vectors for the SVRT-pre-trained network trained using all five dataset sizes (.5k, 1k, 5k, 10k, 15k) and searching over a range of optimal learning rates (1e-4, 1e-5, 1e-6). This led to five-dimensional vector, which we normalize by dividing each entry with the corresponding test accuracy of a baseline ResNet50 trained from scratch. Hence, the normalized vector represent the improvement (ratio larger than 1) or reduction in accuracy (ratio smaller than 1) that results from the pre-training on SVRT for that particular task and training set size. We then calculated the slope vector in $\mathcal{R}^{(23)}$, which we correlated with the corresponding spatial and feature-based attention vectors from Experiment 2.

We found SVRT pre-training to be more correlated with spatial (r = 0.90, p = 4e - 9) than feature-based (r = 0.595, p = 0.002) attention. This suggests that the observed improvements in accuracy derived from spatial attention are more consistent with learning better feature representations compared to feature-based attention.

To summarize, in Experiment 3, we have tried to address the question of learnability of SVRT features vs. rules. We found that using an auxiliary task to pre-train the networks on the SVRT stimuli in order to learn visual representations beforehand provides significant learning advantages to the network compared to a network trained from scratch. We also found a noteworthy correlation between the test accuracy vector of a network initialized with SVRT pre-training and a similar network endowed with spatial attention. This suggests that spatial attention helps discover the abstract rule more so that it helps improve learning good visual representations for the task.

Discussion

The goal of the present study was to shed light on the computational mechanisms underlying visual reasoning using the Synthetic Visual Reasoning Test (SVRT) (Fleuret et al., 2011). The twenty-three binary classification problems in this challenge, which include a variety of same-different and spatial reasoning challenges.

In a first experiment, we systematically evaluated the ability of a battery of N=15 deep convolutional neural networks (ResNets) – varying in depths and trained using different training set sizes – to solve each of the SVRT problems. We found a range of accuracies across all twenty-three tasks, with some tasks being easily learned by shallower networks and relatively small training sets, and some tasks remaining hardly solved with much deeper networks and orders of magnitude more training examples.

Under the assumption that the computational complexity of individual tasks can be well characterized by the pattern of test accuracy across these N neural networks, we formed N-dimensional accuracy vectors for each task and ran a

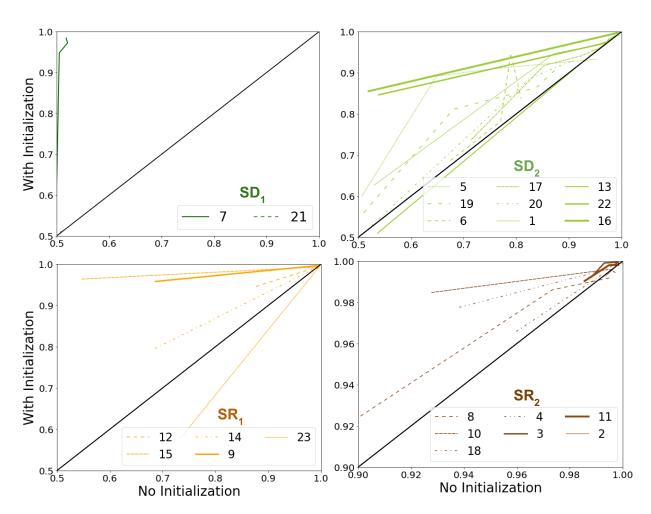


Figure 8: Test accuracies for a baseline ResNet50 trained from scratch ("No initialization") vs. the same architecture pre-trained on an auxiliary task in order to learn visual representations that are already adapted to the SVRT stimuli for different number of training examples. The format is the same as used in Figure 4. Also note that a different axis scale is used for SR_2 to improve visibility.

hierarchical clustering algorithm. The resulting analysis revealed a taxonomy of visual reasoning tasks: beyond two primary clusters corresponding to same-different (SD) vs. spatial relation (SR) judgments, we also identified a finer organization with sub-clusters reflecting the nature and the number of relations used to compose the rules defining the task. Our results are consistent with previous work by Kim et al. (2018) who first identified a dichotomy between SD and SR tasks. Our results also extend prior work (Fleuret et al., 2011; Kim et al., 2018; Yihe et al., 2019) in revealing a finer-level taxonomy of visual reasoning tasks. The accuracy of neural networks is reflected in the number of relationships used to define the basic rules, which is expected, but it deserves closer examination.

Kim et al. (2018) have previously suggested that SD tasks "strain" convolutional neural networks. That is, while it is possible to find a network architecture of sufficient depth (or number of units) that can solve a version of the task up to a number of stimulus configurations (e.g., by forcing all stimuli to be contained within a $\Delta H \times \Delta W$ window), it is relatively easy to render the same task unlearnable by the same network passed a certain number of stimulus configurations (e.g., by increasing the size of the window that contains all stimuli). It is as if these convolutional networks are capable to learn the task if the number of stimulus configurations remains below their memory capacity, and fail beyond that.

Deep convolutional neural networks are universal approximators. That is, they can learn mappings from images to class labels. Depending on the complexity of the mapping one might need an increasing number of hidden units to allow for enough expressiveness of the network; but provided enough units / depth and a sufficient amount of training examples, deep CNNs can learn arbitrary visual reasoning tasks. However, deep CNNs typically lack many of the human cognitive

functions such as attention and working memory. Such functions are likely to provide a critical advantage for a learner to solve some of these tasks (Geman et al., 1992). CNNs might have to rely instead on function approximation which could lead to a less general brute-force solution.

For instance, Linsley et al. (2018b) have shown that contour tracing tasks that can be solved efficiently with a single layer of a recurrent-CNN may require on the order of one hundred processing stages in a non-recurrent-CNN t solve the same task. This ultimately translates into much greater sample efficiency for recurrent-CNNs on natural image segmentation tasks (Linsley et al., 2020). Universal approximators with minimal inductive biases such as multi-layer perceptrons, CNNs and other feedforward or pre-attentive architectures can learn to solve visual reasoning tasks, but they might need a very large number of training examples to properly fit. Hence, beyond simply measuring the accuracy of very deep nets in high data regimes (such as when millions of training examples are available), systematically assessing the performance of neural nets of varying depths and for different training regimes may provide critical information about the complexity of different visual reasoning tasks.

Kim et al. (2018) hypothesized that such straining by convolutional networks is due to their lack of attention mechanisms to allow the explicit binding of image regions to mental objects. A similar point was made by Greff et al. (2020) in the context of the contemporary neural network failure to carve out sensory information into discrete chunks which can then be individually analyzed and compared (see also Tsotsos et al. (2007) for a similar point). Interestingly, this prediction was recently tested using human EEG by Alamia et al. (2021) who showed that indeed the brain activity recorded during SD tasks is compatible with greater attention and working memory demands than SR tasks.

To further assess the role of attention in visual reasoning, we used transformer modules to endow deep CNNs with spatial or feature-based attention. The relative improvements obtained by the CNNs with the two forms of attention varied across tasks. Many tasks reflected a larger improvement for spatial attention, and a smaller number benefited from feature-based attention. Further, we found that the patterns of relative improvements accounted for much of the variance in the space of SVRT tasks derived in Experiment 1. Overall, we found that the requirement for feature-based and spatial attention accounts well for the taxonomy of visual reasoning tasks identified in Experiment 1.

In our third experiment, we studied the learnability of SVRT features vs. rules. We did this by pre-training the neural networks on auxiliary tasks in order to learn SVRT features before training them to learn the abstract rules associated with individual SVRT problems. Our pre-training methods led to networks that learn to solve the SVRT problems better than networks trained from scratch as well as networks that were pre-trained to perform image categorization on the ImageNet dataset. We have also found that such attention processes seem to contribute more to rule learning than to feature learning. For SR_1 sub-cluster we find this type of pre-training to be advantageous in lower training regimes but the benefits rapidly fade away in higher training regimes. In contrast, this pre-training does not allow the tasks from the SD_1 sub-cluster to be learned even with 15k samples – suggesting that the key challenge with these tasks is not to discover good visual representations but rather to discover the rule. This suggests the need for additional mechanisms beyond those implemented in ResNets. This is also consistent with the improvements observed for these tasks with the addition of attention mechanisms.

In summary, this study has compared the computational demands of different visual reasoning tasks. While our focus has been on understanding the computational benefits of attention and feature learning mechanisms, it is clear that additional mechanisms will be required to fully solve all SVRT tasks. These mechanisms are likely to include working memory which is known to play a role in SD tasks (Alamia et al., 2021). Overall, this work illustrates the potential benefits of incorporating brain-like mechanisms in modern neural networks and provides a path forward to achieving human-level visual reasoning.

Acknowledgments

This work was funded by NSF (IIS-1912280) and ONR (N00014-19-1-2029) to TS and ANR (OSCI-DEEP grant ANR-19-NEUC-0004) to RV. Additional support was provided by the ANR-3IA Artificial and Natural Intelligence Toulouse Institute (ANR-19-PI3A-0004), the Center for Computation and Visualization (CCV) and High Performance Computing (HPC) resources from CALMIP (Grant 2016-p20019). We acknowledge the Cloud TPU hardware resources that Google made available via the TensorFlow Research Cloud (TFRC) program as well as computing hardware supported by NIH Office of the Director grant S10OD025181.

References

Alamia, A., Luo, C., Ricci, M., Kim, J., Serre, T., and VanRullen, R. (2021). Differential involvement of eeg oscillatory components in sameness versus spatial-relation visual reasoning tasks. *eNeuro*, 8(1).

Ba, J. L., Kiros, J. R., and Hinton, G. E. (2016). Layer normalization. arXiv preprint arXiv:1607.06450.

- Bello, I., Zoph, B., Le, Q., Vaswani, A., and Shlens, J. (2019). Attention augmented convolutional networks. In 2019 IEEE/CVF International Conference on Computer Vision (ICCV), pages 3285–3294.
- Brady, T. F. and Alvarez, G. A. (2015). Contextual effects in visual working memory reveal hierarchically structured memory representations. *Journal of Vision*, 15(15):6.
- Carion, N., Massa, F., Synnaeve, G., Usunier, N., Kirillov, A., and Zagoruyko, S. (2020a). End-to-End object detection with transformers.
- Carion, N., Massa, F., Synnaeve, G., Usunier, N., Kirillov, A., and Zagoruyko, S. (2020b). End-to-end object detection with transformers. *arXiv* preprint *arXiv*:2005.12872.
- Chen, K., Wang, J., Chen, L. C., Gao, H., Xu, W., and Nevatia, R. (2015). ABC-CNN: an attention based convolutional neural network for visual question answering. *CoRR*, abs / 1511.05960.
- Chen, L., Zhang, H., Xiao, J., Nie, L., Shao, J., Liu, W., and Chua, T.-S. (2017). Sca-cnn: Spatial and channel-wise attention in convolutional networks for image captioning. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 5659–5667.
- Clevenger, P. E. and Hummel, J. E. (2014). Working memory for relations among objects. *Attention, Perception, & Psychophysics*, 76(7):1933–1953.
- Deng, J., Dong, W., Socher, R., Li, L.-J., Li, K., and Fei-Fei, L. (2009). Imagenet: A large-scale hierarchical image database. In 2009 IEEE conference on computer vision and pattern recognition, pages 248–255. IEEE.
- Desimone, R. and Duncan, J. (1995). Neural mechanisms of selective visual attention. *Annual review of neuroscience*, 18(1):193–222.
- Dosovitskiy, A., Beyer, L., Kolesnikov, A., Weissenborn, D., Zhai, X., Unterthiner, T., Dehghani, M., Minderer, M., Heigold, G., Gelly, S., et al. (2020a). An image is worth 16x16 words: Transformers for image recognition at scale. *arXiv preprint arXiv:2010.11929*.
- Dosovitskiy, A., Beyer, L., Kolesnikov, A., Weissenborn, D., Zhai, X., Unterthiner, T., Dehghani, M., Minderer, M., Heigold, G., Gelly, S., Uszkoreit, J., and Houlsby, N. (2020b). An image is worth 16x16 words: Transformers for image recognition at scale.
- Egly, R., Rafal, R., Driver, J., and Starrveveld, Y. (1994). Covert orienting in the split brain reveals hemispheric specialization for object-based attention. *Psychological science*, 5(6):380–383.
- Ellis, K., Solar-Lezama, A., and Tenenbaum, J. (2015). Unsupervised learning by program synthesis. In NIPS.
- Fei-Fei, L., Li, F. F., Iyer, A., Koch, C., and Perona, P. (2007). What do we perceive in a glance of a real-world scene? *J. Vis.*, 7(1):1–29.
- Fleuret, F., Li, T., Dubout, C., Wampler, E. K., Yantis, S., and Geman, D. (2011). Comparing machines and humans on a visual categorization test. *Proceedings of the National Academy of Sciences*, 108(43):17621–17625.
- Funke, C. M., Borowski, J., Stosio, K., Brendel, W., Wallis, T. S. A., and Bethge, M. (2021). Five points to check when comparing visual perception in humans and machines. *Journal of Vision*, 21(3):16.
- Geirhos, R., Jacobsen, J.-H., Michaelis, C., Zemel, R., Brendel, W., Bethge, M., and Wichmann, F. A. (2020). Shortcut learning in deep neural networks. *Nature Machine Intelligence*, 2(11):665–673.
- Geman, D., Geman, S., Hallonquist, N., and Younes, L. (2015). Visual turing test for computer vision systems. *Proc. Natl. Acad. Sci. U. S. A.*, 112(12):3618–3623.
- Geman, S., Bienenstock, E., and Doursat, R. (1992). Neural networks and the bias/variance dilemma. *Neural computation*, 4(1):1–58.
- Golde, M., von Cramon, D. Y., and Schubotz, R. I. (2010). Differential role of anterior prefrontal and premotor cortex in the processing of relational information. *Neuroimage*, 49(3):2890–2900.
- Goyal, A. and Bengio, Y. (2021). Inductive biases for deep learning of higher-level cognition. *arXiv preprint* arXiv:2011.15091.
- Greff, K., van Steenkiste, S., and Schmidhuber, J. (2020). On the binding problem in artificial neural networks. *arXiv* preprint arXiv:2012.05208.
- He, K., Zhang, X., Ren, S., and Sun, J. (2016). Deep residual learning for image recognition. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*.
- Holcombe, A. O., Linares, D., and Vaziri-Pashkam, M. (2011). Perceiving spatial relations via attentional tracking and shifting. *Curr. Biol.*, 21(13):1135–1139.

- Hu, J., Shen, L., and Sun, G. (2018). Squeeze-and-excitation networks. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 7132–7141.
- Kim, J., Ricci, M., and Serre, T. (2018). Not-so-clevr: learning same–different relations strains feedforward neural networks. *Interface focus*, 8(4):20180011.
- Kingma, D. P. and Ba, J. (2014). Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980.
- Kreiman, G. and Serre, T. (2020). Beyond the feedforward sweep: feedback computations in the visual cortex. *Ann. N. Y. Acad. Sci.*
- Kroger, J. K., Sabb, F. W., Fales, C. L., Bookheimer, S. Y., Cohen, M. S., and Holyoak, K. J. (2002). Recruitment of anterior dorsolateral prefrontal cortex in human reasoning: a parametric study of relational complexity. *Cerebral* cortex, 12(5):477–485.
- Linsley, D., Kim, J., Ashok, A., and Serre, T. (2020). Recurrent neural circuits for contour detection. *arXiv* preprint *arXiv*:2010.15314.
- Linsley, D., Scheibler, D., Eberhardt, S., and Serre, T. (2018a). Global-and-local attention networks for visual recognition. *arXiv preprint arXiv:1805.08819*.
- Linsley, D., Shiebler, D., Eberhardt, S., and Serre, T. (2018b). Learning what and where to attend. *arXiv preprint arXiv:1805.08819*.
- Logan, G. D. (1994a). On the ability to inhibit thought and action: A users' guide to the stop signal paradigm. Academic Press.
- Logan, G. D. (1994b). Spatial attention and the apprehension of spatial relations. *Journal of Experimental Psychology: Human Perception and Performance*, 20(5):1015.
- Messina, N., Amato, G., Carrara, F., Gennaro, C., and Falchi, F. (2021). Solving the same-different task with convolutional neural networks. *Pattern Recognition Letters*.
- Moore, C. M., Elsinger, C. L., and Lleras, A. (1994). Visual attention and the apprehension of spatial relations: The case of depth. *J. Exp. Psychol. Hum. Percept. Perform.*, 20(5):1015–1036.
- Ramachandran, P., Parmar, N., Vaswani, A., Bello, I., Levskaya, A., and Shlens, J. (2019). Stand-alone self-attention in vision models. In *Advances in Neural Information Processing Systems (NeurIPS)*.
- Ren, M. and Zemel, R. S. (2016). End-to-end instance segmentation and counting with recurrent attention. *CoRR*, abs / 1605.09410.
- Ricci, M., Cadène, R., and Serre, T. (2021). Same-different conceptualization: a machine vision perspective. *Current Opinion in Behavioral Sciences*, 37:47 55.
- Roelfsema, P. R., Lamme, V. A., and Spekreijse, H. (1998). Object-based attention in the primary visual cortex of the macaque monkey. *Nature*, 395(6700):376–381.
- Rosielle, L. J., Crabb, B. T., and Cooper, E. E. (2002). Attentional coding of categorical relations in scene perception: evidence from the flicker paradigm. *Psychon. Bull. Rev.*, 9(2):319–26.
- Sharma, S., Kiros, R., and Salakhutdinov, R. (2015). Action recognition using visual attention. *arXiv preprint* arXiv:1511.04119.
- Shepard, R. N. and Metzler, J. (1971). Mental rotation of three-dimensional objects. *Science*, 171(3972):701–703.
- Stabinger, S., Piater, J., and Rodríguez-Sánchez, A. (2021). Evaluating the progress of deep learning for visual relational concepts. *arXiv preprint arXiv:2001.10857*.
- Stabinger, S., Rodríguez-Sánchez, A., and Piater, J. (2016a). 25 years of cnns: Can we compare to human abstraction capabilities? In *International Conference on Artificial Neural Networks*, pages 380–387. Springer.
- Stabinger, S., Rodríguez-Sánchez, A., and Piater, J. (2016b). 25 years of cnns: Can we compare to human abstraction capabilities? In Villa, A. E., Masulli, P., and Pons Rivero, A. J., editors, *Artificial Neural Networks and Machine Learning ICANN 2016*, pages 380–387, Cham. Springer International Publishing.
- Stollenga, M. F., Masci, J., Gomez, F., and Schmidhuber, J. (2014). Deep networks with internal selective attention through feedback connections. In *Advances in neural information processing systems*, pages 3545–3553.
- Touvron, H., Cord, M., Douze, M., Massa, F., Sablayrolles, A., and Jégou, H. (2021). Training data-efficient image transformers & distillation through attention.
- Tsotsos, J. K., Rodriguez-Sanchez, A. J., Rothenstein, A. L., and Simine, E. (2007). Different binding strategies for the different stages of visual recognition. In Mele, F., Ramella, G., Santillo, S., and Ventriglia, F., editors, *Advances in Brain, Vision, and Artificial Intelligence*, pages 150–160, Berlin, Heidelberg. Springer Berlin Heidelberg.

- Van Der Ham, I. J. M., Duijndam, M. J. A., Raemaekers, M., van Wezel, R. J. A., Oleksiak, A., and Postma, A. (2012). Retinotopic mapping of categorical and coordinate spatial relation processing in early visual cortex. *PLoS One*, 7(6):1–8.
- Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A. N., Kaiser, Ł., and Polosukhin, I. (2017). Attention is all you need. In *Advances in Neural Information Processing Systems (NIPS)*.
- Woo, S., Park, J., Lee, J.-Y., and So Kweon, I. (2018). Cbam: Convolutional block attention module. In *Proceedings of the European conference on computer vision (ECCV)*, pages 3–19.
- Xu, H. and Saenko, K. (2015). Ask, attend and answer: Exploring question-guided spatial attention for visual question answering. *CoRR*, abs / 1511.05234.
- Yang, Z., He, X., Gao, J., Deng, L., and Smola, A. (2016). Stacked attention networks for image question answering. In 2016 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pages 21–29.
- Yihe, L., Lowe, S. C., Lewis, P. A., and van Rossum, M. C. (2019). Program synthesis performance constrained by non-linear spatial relations in synthetic visual reasoning test. *arXiv* preprint arXiv:1911.07721.
- Zhu, X., Su, W., Lu, L., Li, B., Wang, X., and Dai, J. (2020). Deformable DETR: Deformable transformers for End-to-End object detection.

Supplementary Information

S1 ResNet50 with attention

Here we detail how we adapt the attention module originally developed for natural language processing in (Vaswani et al., 2017) and insert it in an already existing convolutional network architecture such as ResNet50. Similar adaptations can be found in recent works (Carion et al., 2020b; Ramachandran et al., 2019).

Briefly, transformer architectures are encoder-decoder-based architectures consisting of at least one self-attention module followed by a feedforward layer. Here, we simplified the standard transformer architecture where we picked only one encoder layer without any feedforward layer and removed the decoder part as routinely done in vision (Dosovitskiy et al., 2020a; Touvron et al., 2021; Ramachandran et al., 2019; Bello et al., 2019). The motivation for removing these layers is that we want to evaluate the effect of attention on the basis of its ability to modulate the neural activity as opposed to simply adding more depth to the neural representations. Our attention modules use *key, query and values* variables to learn to attend to either spatial or feature locations within a given layer of processing.

Spatial Attention Module (SAM) Our first attention module takes a features map $X \in \mathbb{R}^{d_C \times d_H \times d_W}$ as input, where d_C , d_H , and d_W respectively refer to the number of channels, height and width of the map, and outputs a features map Y of the same dimensions. We flatten the spatial dimensions to obtain $X' \in \mathbb{R}^{d_C \times d_N}$, where $d_N = d_H * d_W$, and we apply the original multi-head self-attention module from Vaswani et al. (2017) as follows.

We first apply independent linear mappings of the input X' to obtain three features maps of dimensions $\mathbb{R}^{d \times d_N}$ for each attention head from a total of n_H heads. For the i^{th} head, these maps are known as the query Q_i , the key K_i and the value V_i , and are obtained such as:

$$Q_i = W_i^Q.X'$$

$$K_i = W_i^K.X'$$

$$V_i = W_i^V.X'$$

The mappings are parametrized by three matrices W_i^Q , W_i^K and W_i^V of dimensions $\mathbb{R}^{d\times d_C}$ for each head. The symbol . denotes a matrix multiplication.

Then, we apply the scaled dot-product attention (Vaswani et al., 2017) to obtain n_H attention heads of dimensions $\mathbb{R}^{d \times d_N}$ such as:

$$H_i = SoftMax(\frac{Q_i.K_i^T}{\sqrt{d}})V_i \tag{1}$$

After, we concatenate all attention heads along the first dimension and apply a linear mapping to obtain $Y' \in \mathbb{R}^{d_C \times d_N}$ such as:

$$Z = W^O.Concat(H_1, ..., H_{n_H})$$
(2)

The mapping is parametrized by the matrix $W^O \in \mathbb{R}^{d_C \times d}$.

As commonly done, we have a residual connection before applying a layer normalization (Ba et al., 2016) such as:

$$Y' = LayerNorm(Z + X') \tag{3}$$

Finally, we unflatten Y' to obtain $Y \in \mathbb{R}^{d_C \times d_H \times d_W}$.

We obtain the best results with a representation space of 512 dimensions (d = 512) and four attention heads ($n_H = 4$).

Features-based Attention Module (FBAM) Our second attention module is simply obtained by transposing the channel dimension with the spatial dimensions before applying the same transformations. In other words, we transpose the input X' into $\mathbb{R}^{d_N \times d_C}$ and transpose the output Y' back into $\mathbb{R}^{d_C \times d_N}$. While SAM models an attention over the $d_H * d_W$ regions that compose the input features map, FBAM models an attention over the d_C features channels.

We obtain the best results with a representation space of 196 dimensions (d = 196) and one attention head ($n_H = 1$).

Insertion of attention modules There are five convolutional blocks in the ResNet-50 architecture. We insert two blocks of either our SAM or FBAM after the third and fourth blocks as seen in Figure S1.

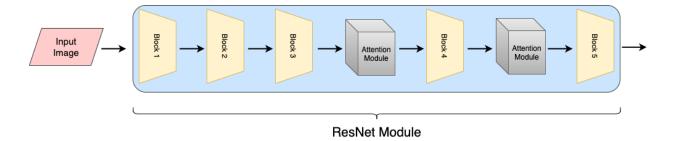


Figure S1: Insertion of one of our attention modules into ResNet-50.

Table 2: Each cell represents attempts participants took to solve seven consecutive correct categorization. Here, row and column represents $task\ number$ and $participant\ number$. Entries containing "X" indicate that the participant failed to solve the problem, and those cells are not included in the marginal means. (Fleuret et al., 2011)

	Participant No.														M	E-21						
Task No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Mean	Fail
1	1	12	1	2	8	8	1	1	X	1	14	1	4	1	1	1	2	1	1	1	3.26	1
2	3	1	2	2	10	19	4	4	14	3	2	3	21	1	1	5	3	2	22	9	6.55	0
3	7	1	3	1	4	3	1	1	7	1	6	1	1	1	4	1	1	1	4	2	2.55	0
4	1	6	7	1	1	3	1	1	1	1	3	1	1	2	1	1	7	5	7	1	2.6	0
5	7	X	1	21	8	3	1	5	X	1	X	9	13	1	6	2	X	8	1	7	5.88	4
6	X	20	X	X	27	25	12	26	X	X	3	X	X	X	4	16	X	X	X	X	16.63	12
7	1	X	1	X	13	8	4	14	X	3	8	12	7	X	1	6	1	1	14	9	6.44	4
8	7	6	1	14	4	14	1	5	1	4	8	1	1	1	13	5	3	7	4	1	5.05	0
9	4	24	1	16	3	1	1	13	X	X	4	6	X	2	7	1	3	1	5	1	5.47	3
10	1	8	2	2	4	1	3	5	X	4	1	2	16	4	4	2	1	1	4	3	3.58	1
11	4	2	3	1	3	1	4	8	1	2	1	1	1	1	1	5	2	1	1	1	2.2	0
12	1	2	8	1	9	4	8	4	1	7	25	2	5	2	X	2	5	X	4	1	5.06	2
13	1	20	5	14	X	3	ĭ	13	7	10	1	13	9	5	X	3	3	2	X	1	6.53	3
14	4	4	1	1	3	10	2	X	12	14	1	19	1	3	1	1	4	8	1	2	4.84	1
15	1	X	1	2	2	1	1	1	X	5	1	2	4	1	1	18	10	3	2	1	3.17	2
16	12	18	7	X	X	2	2	14	X	X	28	9	13	X	22	10	X	X	X	X	12.45	9
17	14	X	6	5	2	X	21	X	X	22	X	14	X	X	X	X	13	8	28	1	12.18	9
18	5	17	2	X	27	5	5	1	X	2	X	7	19	4	1	1	5	1	1	2	6.18	3
19	2	10	1	11	1	3	5	11	8	2	4	2	17	1	4	4	1	6	1	X	4.95	1
20	14	7	4	5	i	8	3	1	X	18	9	16	3	i	6	i	2	ĩ	15	1	6.11	1
21	6	X	i	X	i	X	23	X	X	21	28	7	26	7	15	2	17	X	16	X	13.08	7
22	1	9	14	î	i	4	1	5	21	2	1	2	5	i	6	ī	4	î	1	6	4.35	Ó
23	1	í	7	22	i	i	2	1	6	21	2	5	4	6	4	3	1	i	6	8	5.15	0
Mean	4.45	9.33	3.59	6.78	6.33	6.05	4.65	6.7	7.18	7.2	7.5	6.14	8.55	2.37	5.15	4.14	4.4	3.11	6.9	3.05		
No of Fails	1	5	1	5	2	2	0	3	12	3	3	1	3	4	3	1	3	4	3	4		

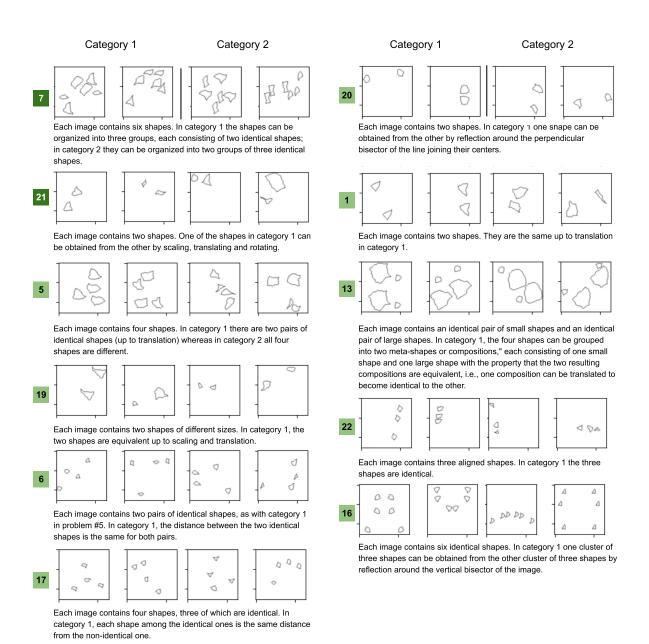


Figure S2: Sample images for Same Different (SD) tasks

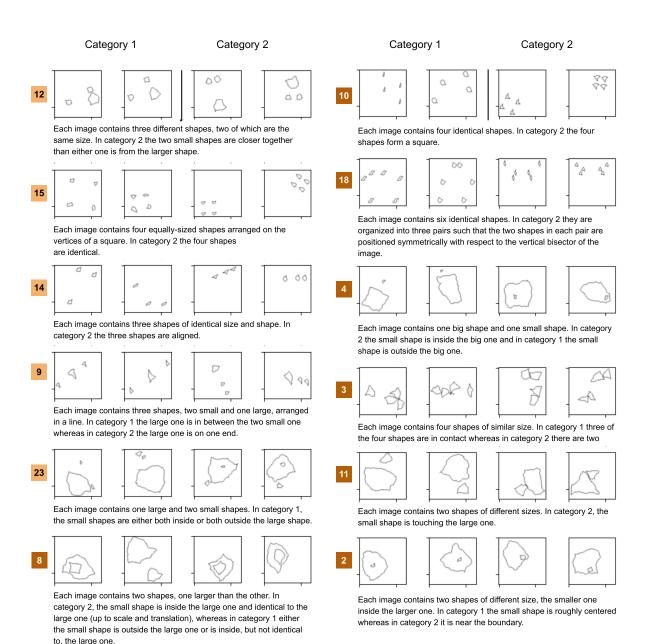


Figure S3: Sample images for Spatial Relation (SR) tasks

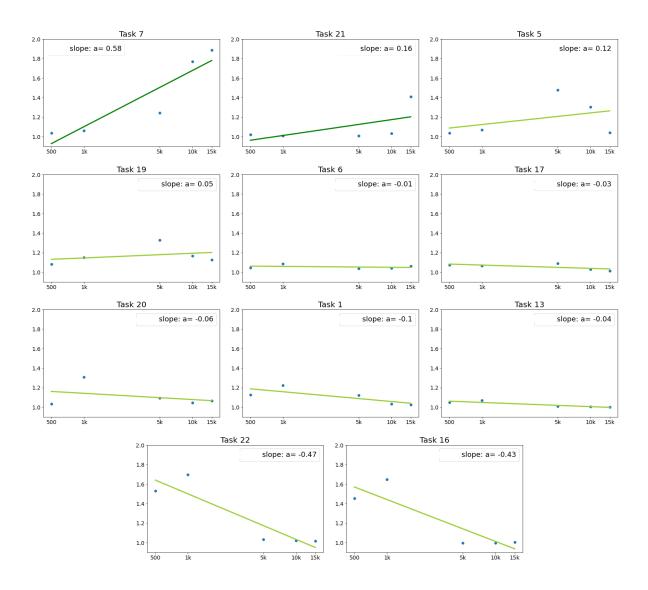


Figure S4: Slope attained by linear fitting of points obtained after taking the ratio of each of the network with spatial attention module and the test accuracy of a ResNet50 for each task and training condition for Same Different (SD) tasks

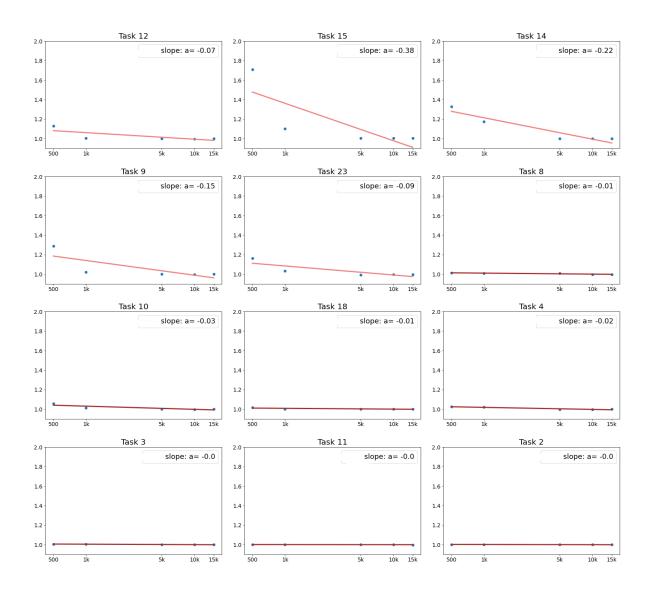


Figure S5: Slope attained by linear fitting of points obtained after taking the ratio of each of the network with spatial attention module and the test accuracy of a ResNet50 for each task and training condition for Spatial Relation (SR) tasks

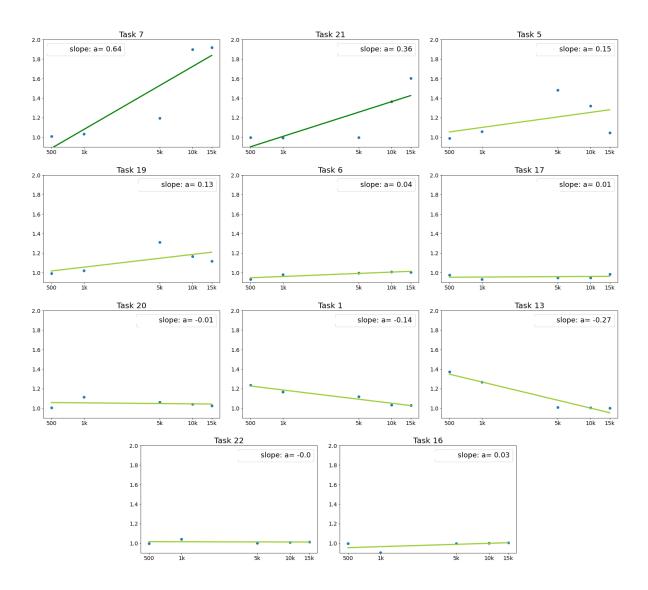


Figure S6: Slope attained by linear fitting of points obtained after taking the ratio of each of the network with feature-based attention module and the test accuracy of a ResNet50 for each task and training condition for Same Different (SD) tasks

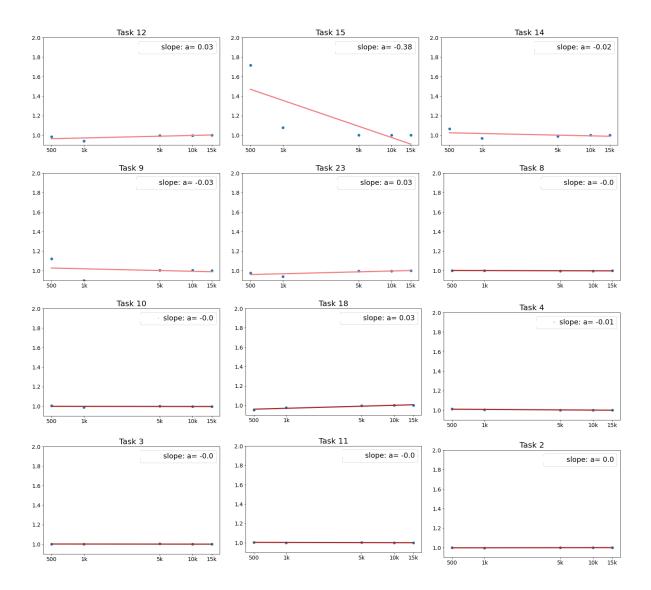


Figure S7: Slope attained by linear fitting of points obtained after taking the ratio of each of the network with feature-based attention module and the test accuracy of a ResNet50 for each task and training condition for Spatial Relation (SR) tasks

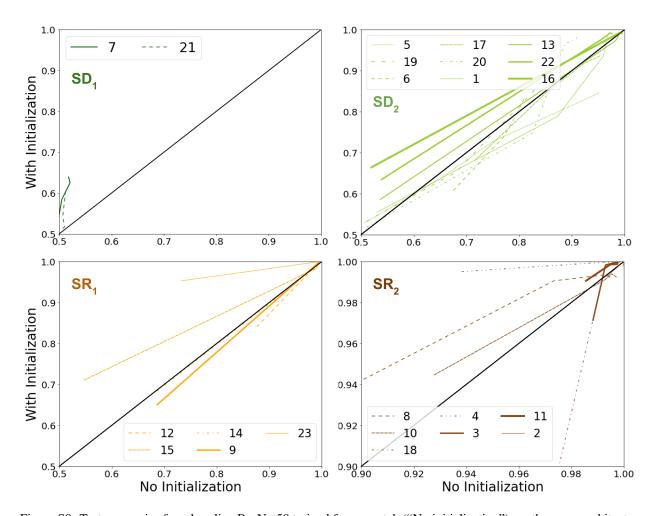


Figure S8: Test accuracies for a baseline ResNet50 trained from scratch ("No initialization") vs. the same architecture pre-trained on Imagenet data for different number of training examples. The format is the same as used in Figure 4. Also note that a different axis scale is used for SR_2 to improve visibility.